during its optimization process. We are not reporting these user-hours because of the difficulty in assigning an accurate metric to them.

Conclusions

The DBE method provides a fast and flexible way to evaluate the boundary conditions and performance index for the approximate translunar problem. No reprogramming is required to change specified system parameters, such as the initial thrust-to-weight ratio, and any interpolation errors associated with the tabular method are eliminated.

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Nonlinear Autopilot Design for Bank-to-Turn Steering of a Flight Vehicle

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I. Introduction

In bank-to-turn (BTT) steering of a flight vehicle, the control system continuously banks the flight vehicle to minimize the side-slip angle and, hence, the asymmetry of its vortex wake. Consequently, the flight vehicle can maneuver at a higher angle of attack, thus increasing the lift capability and accomplishing a turn in a shorter time. Although the BTT steering offers much theoretical improvements in maneuverability, practical flight vehicles that use this steering are still not able to achieve the desired results. This is because of inherent limitations in the autopilot designs. In recent years, nonlinear autopilot designs based on dynamic inversion have been developed.[1–5] In this Note, we present a nonlinear control law that takes into consideration the full nonlinear dynamics of the flight vehicle. The approach in autopilot design is based on the differential geometric control theory popularized by Brockett.[6] We show that the nonlinear coupling between the kinematics and dynamics can essentially be canceled using feedback and kinematic inversion in the control law. The use of the kinematic inversion essentially removed the nonminimum phase zero associated with the airframe dynamics. The nonlinear control law developed requires only measurements of the roll angle, incidence angle, side-slip angle, and body angular rates.

II. Problem Formulation

We present a new formulation of the dynamical equations of motion. The following assumptions are made in the development of the nonlinear autopilot.

Assumption 1. The control surface effectiveness terms are small compared to the aerodynamics derivatives and damping terms.

Assumption 2. The airframe is rigid.

Let α denote the incidence angle and β the side-slip angle. The evolution of incidence and side-slip angles is dependent on the body angular rates. Thus, these angles can be indirectly controlled via the body rates. We define a new set of output variables,

$$y_1 := \phi \tag{1}$$

$$y_2 := \tan\alpha \tag{2}$$

$$y_3 := \tan\beta \tag{3}$$

where ϕ is the roll angle. Given the desired roll angle and the desired incidence and side-slip angles, we can compute the desired output directly using the definitions. Let $\boldsymbol{y} := [y_1, y_2, y_3]^T$; then, the time derivative of the output vector is

$$\dot{\boldsymbol{y}} = \boldsymbol{C}(\boldsymbol{y})\boldsymbol{x} \tag{4}$$

where

$$\boldsymbol{x} := \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{5}$$

$$\boldsymbol{C}(\boldsymbol{y}) := \begin{bmatrix} 1 & 0 & 0 \\ -y_3 & 1+y_2^2 & -y_2y_3 \\ y_2 & y_2y_3 & -\left(1+y_3^2\right) \end{bmatrix} \tag{6}$$

Suppose the system input variables are the equivalent aileron, elevator, and rudder command, denoted by $\boldsymbol{u}$. The evolution of the state vector $\boldsymbol{x}$ is given by Euler's equation and the equations describing the aerodynamic moments

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} \tag{7}$$

where $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$ and $\boldsymbol{g}(\boldsymbol{x})$ are functions of the aerodynamics and kinematics of the flight vehicle. The equations of motion of the system are completely described by Eqs. (4) and (7). Moreover, the kinematics and dynamics of these equations are tightly coupled and highly nonlinear.

III. Nonlinear Autopilot

We develop the nonlinear autopilot. The nonlinear differential geometric approach we adopt was first popularized by Brockett.[6] We rewrite the equations of motion as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} \tag{8}$$

$$\dot{\boldsymbol{y}} = \boldsymbol{C}(\boldsymbol{y})\boldsymbol{x} \tag{9}$$

Since the control law does not appear in the output, Eq. (9), we differentiate the output with respect to time, and after some simplification we obtain

$$\ddot{\boldsymbol{y}} = \frac{\partial \boldsymbol{C}(\boldsymbol{y})\boldsymbol{x}}{\partial \boldsymbol{y}}\boldsymbol{C}(\boldsymbol{y})\boldsymbol{x} + \boldsymbol{C}(\boldsymbol{y})\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) + \boldsymbol{C}(\boldsymbol{y})\boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} \tag{10}$$

Thus a nonlinear control law, which completely decouples the output equation, can be written as

$$\boldsymbol{u} = [\boldsymbol{C}(\boldsymbol{y})\boldsymbol{g}(\boldsymbol{x})]^{-1}\left[-\frac{\partial \boldsymbol{C}(\boldsymbol{y})\boldsymbol{x}}{\partial \boldsymbol{y}}\boldsymbol{C}(\boldsymbol{y})\boldsymbol{x} - \boldsymbol{C}(\boldsymbol{y})\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) + \boldsymbol{\nu}\right] \tag{11}$$

where $\boldsymbol{\nu} \in \boldsymbol{R}^3$ is an auxiliary input. The closed-loop equation of motion using the control law is given by

$$\ddot{\boldsymbol{y}} = \boldsymbol{\nu} \tag{12}$$

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Hence, the resulting system is linear and decoupled. Suppose the desired output is denoted by y_d; then the auxiliary input, which gives an exponential response, can be written as

$$\nu = -[\lambda_1 \dot{y} + \lambda_2(y - y_d)] \tag{13}$$

Let $y_e := y - y_d$; then the closed-loop error dynamics of the flight control system is given by

$$\ddot{y}_e + \lambda_1 \dot{y}_e + \lambda_2 y_e = 0 \tag{14}$$

Clearly, the error dynamics decays exponentially. If the preceding control law is used directly, then measurements of the time derivative of the output need to be given. In practice, however, the time derivatives of the incidence and side-slip angles are not easy to obtain. Hence, a more practical approach is to estimate the time derivative of the output using the output equation given in Eq. (9). Thus, the nonlinear control law can be written as

$$u = -[C(y)g(x)]^{-1}\left[\frac{\partial C(y)x}{\partial y}C(y)x + C(y)f(x,y) + \lambda_1 C(y)x + \lambda_2(y - y_d)\right] \tag{15}$$

where

$$\dot{y} = C(y)x = \begin{bmatrix} p \\ -y_3 p + \left(1 + y_2^2\right)q - y_2 y_3 r \\ y_2 p + y_2 y_3 q - \left(1 + y_3^2\right) r \end{bmatrix} \tag{16}$$

$$\frac{\partial C(y)x}{\partial y} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2y_2 q - y_3 r & -p - y_2 r \\ 0 & p + y_3 q & y_2 q - 2y_3 r \end{bmatrix} \tag{17}$$

Remarks.

1) The measurements required for the nonlinear control law are the state variables $x := \omega$ and the output variables $y := [\phi, \tan\alpha, \tan\beta]^T$. In practice, the time derivative of the output vector y need not be measured since these terms can be calculated analytically.

2) The controller gain matrices λ_1 and λ_2 can be selected to place the closed-loop poles of each control loop at any desired location. In practice, the bandwidth of the roll and yaw loops are chosen to be higher than the pitch loop.

3) The computation of the vector f in the nonlinear controller is dependent on the aerodynamic coefficients. These aerodynamic coefficients can be tabulated according to the current flight conditions, that is, the Mach number, altitude, and angle of attack.

4) The matrix inversion required in the controller is always possible since the matrix C represents a kinematic relation and is always nonsingular. The control input coefficient matrix g must also be nonsingular; otherwise, the system would not be controllable. Hence, there is no singularity problem in the computation of the nonlinear control law.

IV. Simulation Results

We present computer simulations of a generic flight vehicle. The aerodynamic coefficients are obtained from Nelson[7] at 0.8 Mach and an altitude of 4.5 km. The simulation model of the flight vehicle is a full six-degree-of-freedom nonlinear model. An actuator model with both position and velocity limits is also included in the simulation.

The nonlinear control gains are $\lambda_1 = [8.0, 8.0, 2.0]^T$ and $\lambda_2 = [16.0, 12.0, 16.0]^T$. This is equivalent to setting the bandwidth of the roll and the side-slip control loop to 4 rad/s and the incidence control loop to 3.46 rad/s.

An arbitrary initial condition is selected with all body angles and control surfaces set to zero. A step input of 1.2 rad (69 deg) bank angle and 40 ms^{-2} normal acceleration is commanded. The response of the flight vehicle is shown in Figs. 1 and 2 for initial vehicle velocity ranging from Mach 0.6 to Mach 0.9. Figure 1 shows the roll position of the flight vehicle. Note that the desired bank angle is achieved in 2 s. The normal acceleration shown in Fig. 2 indicates that the desired acceleration is achieved in 3 s.

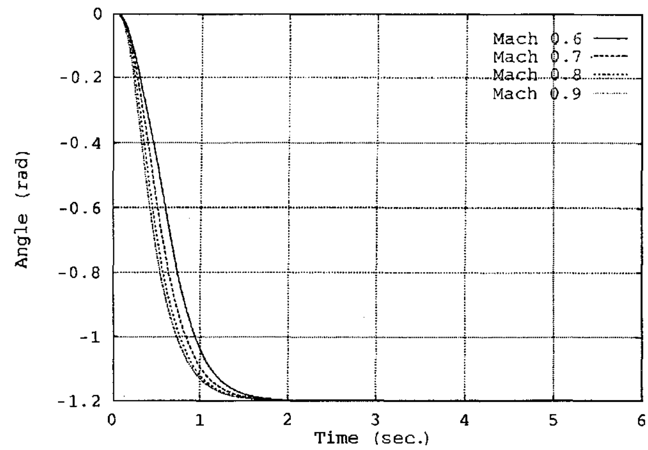

Fig. 1 Body axis roll angle.

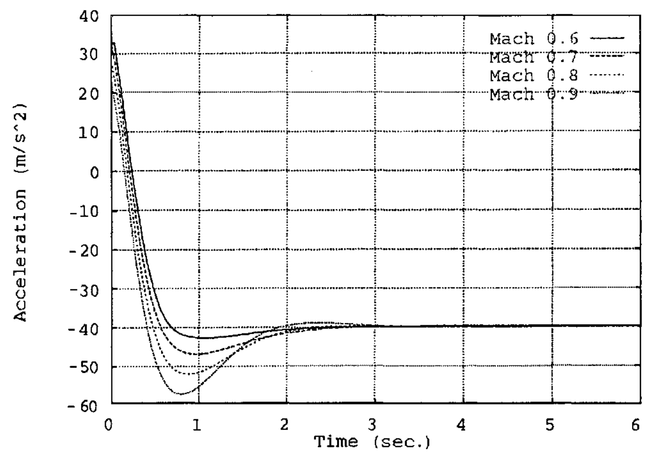

Fig. 2 Normal acceleration.

V. Conclusions

A nonlinear autopilot design for a flight vehicle has been presented. The controller requires only measurements of the incidence angle, side-slip angle, roll angle, and body angular rates. Essential to the successful implementation of this design is an accurate knowledge of the aerodynamic coefficients, since these aerodynamic coefficients vary with the Mach number, altitude, and angle of attack. The computation of the nonlinear controller has to be scheduled according to the flight conditions. This is similar to the gain-scheduling approach used in classical design. Unlike classical design, however, the control synthesis is valid throughout the entire flight envelope of the flight vehicle.

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